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ABSTRACT

This study examined the effects of a specified set of activities on the ability of educable mentally retarded (EMR) children to use the transitive property and conserve the three relations "same number as," "more than," and "fewer than." Thirteen EMR students, aged 12 to 15, were taught eleven lessons on establishing and conserving these relations, and on the use of the transitive property. Three tests were given as pretests and readministered later as posttests and retention tests. Analyses of numbers reaching criterion showed that significant improvement occurred during the instructional period and was maintained thereafter. (MM)

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# THE EFFECTS OF SELECTED EXPERIENCES ON THE ABILITY OF EMR CHILDREN TO CONSERVE AND USE THE TRANSITIVE PROPERTY OF THREE MATCHING RELATIONS: A FEASIBILITY STUDY*

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The purpose of this study was to investigate the effects of a specified set of activities on the ability of educable mentally retarded (EMR) children to use the transitive property and conserve the three relations, "same number as," "more than," and "fewer than."

Piaget has observed four main stages in the development of cognitive operations. The first stage is called the sensorimotor period which lasts for approximately the first 18 months of life. This is a period of action without mental representation, and it is during this stage that object permanency is acquired (Sinclair, 1971, pp. 2-3). During the second stage of preoperational representation, the child builds up a semilogic, a logic of one-way mappings. While this semilogic is incomplete, it is necessary for the child to pass through this stage to acquire reversibility and concrete operations (Sinclair, 1971, pp. 4-5). Around the age of six or seven the first concrete operations appear. Concrete operations refers to operations that are logical, mental actions that are physically possible on real objects, that is, the child can perform the mental operations regardless of whether the objects are actually manipulated or whether the child is asked verbal questions(Sinclair, 1971, p. 5). Conservation is viewed by Piaget (1964, p.9) as the psychological criterion for the presence of reversible operations. In the stage of formal operations from about twelve years of age, the individual is able to perform propositional logic on verbally stated hypotheses, regardless of whether the hypotheses are true (Sinclair, 1971, p. 9).

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Piaget (1970, pp. 27-30) has postulated mental structures (called grouping structures) which he believes is a model for the systems of concrete operations. He (Piaget, 1970, p. 38) has found that development of the natural numbers for the child, is a synthesis of structures of class inclusions and structures of order relations (seriations or asymmetrical relations). Composition in the structure of order relations (e.g., R) is made possible by the transitive property: if aRb and bRc, then aRc (Sinclair, 1971, p. 9).

Van Engen (1971, pp. 37-41) does not agree with Piaget's analysis of number concepts. He maintains that, from a mathematical point of view, number involves set equivalence. Equivalence is an equivalence relation and has the reflexive, symmetric and transitive properties.

Thus, from a psychological or mathematical point of view, the transitive property of order relations and equivalence relations is involved in the development of a concept of the natural numbers. From a psychological point of view, conservation is seen to be necessary for a concept of number. Empirical evidence confirms that conservation is related to performance in mathematics.

Steffe (1966, pp. 46-47) found that first grade children in the lowest level of conservation of numerousness scored significantly lower than three other levels on solving arithmetic addition problems. The subjects were partitioned into 12 groups, 4 levels of conservation of numerousness by 3 IQ groups. The children who scored the highest on addition problems were in the top three levels of conservation and in the top two IQ groups, together with the children in the lowest IQ, highest level group. The children in the lowest level, lowest IQ group scored the lowest on addition problems, and the remaining four cells made up a middle performance category. In a similar study using subtraction problems, Le Blanc (1968, p. 156) found that, with one exception, the mean

performances of the three IQ groups in the highest level (of conservation) were higher than mean performances of the three IQ groups in the second level, and so on for all four levels. He concluded that conservation is more closely related to success in solving subtraction problems than is IQ.

Piaget does not assert fixed ages at which a stage of operation begins in individual children. What is considered invariant is that operating at one level is prerequisite to the next stage, and thus, the order of the stages is fixed. Empirical evidence confirms this hypothesis for educable mentally retarded subjects.

Quick (1966) studied the performance of 80 EMR children on a number concept test. The test involved five Piagetian tasks: conservation of "same number as" based on counting, conservation of one-to-one correspondence, seriation, and the coordination of cardinal and ordinal numbers. Quick's subjects ranged in age from 6;5* to 13;10 and in mental age (MA) from 4;6 to 8;6 as measured by a standardized test.

Quick (1966, pp. 69-70) found a significant correlation between the total score on the number concept test and MA. When be blocked on mental age, he found that subjects with MA less than six gave significantly more global type responses than subjects with MA greater than six. Quick's most (educationally) significant finding was that EMR children experience the same stages of mental development as normal subjects. There was a MA lag in that the subjects in the study had not become concrete operational by MA 8;6.

Woodward (1961) studied 50 adults and 44 children who were institutionalized in homes for the mentally subnormal or in a hospital. The four types of tasks involved establishing and conserving a one-to-one correspondence, equalizing

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*6 years, 5 months.

unequal sets, seriating, and conserving liquid.

Woodward (1961, pp. 256-259) found that most of the subjects scored consistently across all four types of tasks at either the intuitive or concrete operational level. There was a substantial number (38) of subjects who did not understand the instructions for equalizing unequal sets nor conservation of liquid, and who scored at an intuitive level on the other two tasks. Woodward found that IQ's of 50 or more occurred significantly more often among subjects who were operational on at least one task. She concluded that mentally subnormal adults and children make responses similar to normal children of four to seven years of age when dealing with Piagetian number concept problems.

Many EMR children lack conservation and the transitive property of certain relations. Results of empirical studies suggest that this may be a serious handicap in their performance in solving mathematical problems. The question, which is raised, is whether these pupils can acquire the ability to conserve and use the transitive property through specified experiences.

## Method

### Subjects

Thirteen pupils, who were classified as EMR students by the Clarke County School System of Athens, Georgia, and who were enrolled in special classes, served as subjects. The students ranged in age from 12:8 to 15:6. The only standardized test data available, much of which was four or five years old, indicated that the approximate IQ range of the sample was 45 to 75.

### Tests

Three tests were constructed by the investigators. The purpose of the Language Test (LT) was to determine the ability of the students to use the relational terminology to accurately describe relations between two sets of objects. In each of the six items which were constructed, the subject was instructed to match two sets of objects, say A and B. He was then asked three standard questions, "Are there the same number of a's as b's?" "Are there more a's than b's?" and "Are there fewer a's than b's?" and given the opportunity to respond to each question.

There were three items of the Conservation of Matching Relations Test (CMRT) with one item in which each relation held. In each item the subject was instructed to match two sets of objects and was asked the three standard questions. The examiner then rearranged one set of objects to induce a perceptual bias against the relation which existed and repeated the three standard questions.

The Transitivity of Matching Relations Test (TMRT) contained one item on the transitive property of each of the three relations. For a description of an item, consider the item on which there were fewer a's than b's and fewer b's than c's. The subject was instructed to match sets A and B. He was then

asked, "Are there fewer a's than b's?" After the response, he was instructed to match the B and C. The examiner asked "Are there fewer b's than c's?" Following the response, the examiner asked the three standard questions relative to A and C which were not compared. In some tests at the time the transitive inference was required, sets A and C were arranged to induce a perceptual bias against the correct answers to the standard questions. In other cases, sets A and C were screened from the subject's view by placing the objects in a cup following the respective comparisons with set B.

Each material set of the tests consisted of from six to ten objects such as checkers, tiles, cutout stars, or colored wooden discs.

Unit of Instruction

The treatment consisted of eleven lessons, one on establishing relations, five on conserving the relations, and five on using the transitive property of the relations. Instructional materials consisted of sets of physical objects such as small sticks, pebbles, geometric shape cutouts, and pictorial materials. In some cases each student had sets of manipulative materials and performed his own manipulations and transformations. In other lessons the teacher manipulated the materials on the overhead projector, and students responded to appropriate questions. Pictorial materials consisted of cutouts of baseball players and bats and transparencies comparing sets of cars, drivers, and seat belts. The number of objects used in a comparison ranged from seven to about 10.

In the conservation lessons, after two sets were compared, one set either underwent a transformation or was screened from the subjects' view, and the students were asked what relation held. The principle of reversibility was involved by returning objects to the original position of matching the

sets following a relation preserving transformation. Adding or removing objects from one of the sets was used to change the relation between two sets. In the case of the relation, "the same number as," the rule, "rearranging does not change the number of objects, but removing an object does," was given in some conservation lessons.

The method of the lessons on the transitive property was the pupils' empirically determining the relation between A and C in the transitive paradigm, if A is related to B and B is related to C, then A is related to C. After establishing the relation between A and B and between B and C in each example, the pupils were asked to make a conjecture about the relation between A and C before comparing A and C. The sets A and C were either placed in a configuration which was biased against the correct conclusion or screened from the students' view at the time of the conjecture.

Procedure

The tests were given by the investigators in one-to-one interviews. Pretests were given in January, 1971. Each pretesting sessions included a brief training session to insure that the subjects could follow the commands and apply the relational terminology to appropriate situations. Each of these training sessions involved four to seven situations, depending upon the amount of practice the student appeared to need. Following the training session, the Language Test was given. The criterion for the LT was set at one item for each relation. If a subject failed an item, he was given the second item for that relation. The conservation test, CMRT, and the transitivity test, TMRT, followed the LT.

Instruction was given during the two regular mathematics classes by the classroom teacher. The investigators observed the classroom teacher

during the lessons to insure that the teacher followed closely the prescribed lessons and to make sure that the teacher was consistent in his presentation to both groups of students.

In the Spring of 1971, following the completion of instruction, the LT, CIRT and TIRT were given as posttests. The same tests were again given as retention tests 28 days after the posttest for the first class and 16 days after the posttest for the second group. The test differed only in that in the pretests and posttests the objects of the TIRT were arranged to be perceptually biased against the correct solution, and in the retention test the objects were screened at the time of the transitive inference.

In order to pass an item, a subject had to answer correctly each question of the item. The criterion for passing the conservation or transitivity test was set at two of the three items. The probability of meeting the criterion on a test by guessing was less than .043.

Hypotheses and analyses

The hypotheses examined in the study were that there would be a significant increase in the number of subjects who:

(1) conserved the relations "same number as," "more than," and "fewer than," as determined by the pretest and posttest;

(2) used the transitive property of the same relations, as determined by the pretest and posttest;

(3) conserved the specified relations, as determined by the pretest and retention test

(4) used the transitive property of the relations, as determined by the pretest and retention test.

The frequencies of pupils meeting the criteria and not meeting the criteria on the pretest versus the posttest or retention test are presented in 2 X 2 tables. The binomial test was performed on the data in each table to test the null hypothesis that the probability of a pupil changing from fail to pass was equal to the probability of a pupil changing from pass to fail (Siegel, 1956, pp. 36-38, 66-67).

## Results

One pupil failed to reach criterion on the language at any testing. He was consequently dropped from the analyses. One student was absent on the day of the posttest and a different student was absent on the day of the retention test. Thus data are reported for eleven students in each table.

The results of the pupils' performances on pretests and posttests are presented for conservation in Table 1 and for transitivity in Table 2. It may be observed that seven students attained conservation on the posttest who did not on the pretest, and no students changed in the opposite direction. The probability of this occurring under the null hypotheses is less than .01 ($p < .01$). There were four students who failed to meet the criterion on the transitive property on the pretest, and who achieved the criterion on the posttest. No students were changed in the opposite direction. This occurs under the null hypothesis with $p < .07$.

Table 1

Frequency of Pupils Achieving the Criterion on Conservation: Pretest Versus Posttest

| | | Posttest | |
|---|---|---|---|
| | | Not Criterion | Criterion |
| Pretest | Criterion | 0 | 3 |
| | Not Criterion | 1 | 7 |

Table 2

Frequency of Pupils Achieving the Criterion on Transitivity: Pretest Versus Posttest

| | | Posttest | |
|---|---|---|---|
| | | Not Criterion | Criterion |
| Pretest | Criterion | 0 | 4 |
| | Not Criterion | 3 | 4 |

Tables 3 and 4 contain the results of the pretests and retention tests for conservation and transitivity. It may be noted from Table 3 that seven pupils reached the criterion on the conservation retention test who did not meet the criterion on the corresponding pretest. Five pupils changed to criterion on the transitivity retention test. No pupils changed in the opposite direction on either the conservation or transitivity retention test. Under the null hypothesis, the above frequencies may occur with $p < .01$ for conservation and $p < .04$ for transitivity.

In no case did a student change from attaining the criterion on a pretest to failing to attain the criterion on a later test. In each of the four uses of the binomial test, the probability, that the frequencies were obtained under the null hypothesis, was sufficiently small to reject the null hypothesis. Thus, the four hypotheses of this study were accepted.

Table 3

Frequency of Pupils Achieving the Criterion on Conservation: Pretest Versus Retention Test

| | | Retention Test | |
|---|---|---|---|
| | | Not Criterion | Criterion |
| Pretest | Criterion | 0 | 4 |
| | Not Criterion | 0 | 7 |

Table 4

Frequency of Pupils Achieving the Criterion on Transitivity: Pretest Versus Retention Test

| | | Retention Test | |
|---|---|---|---|
| | | Not Criterion | Criterion |
| Pretest | Criterion | 0 | 3 |
| | Not Criterion | 3 | 5 |

In addition to improvement between pretesting and later tests, another finding worthy of note is the status of the pupils before the lessons. Of the 12 children who attained the criterion on relations, 7 attained criterion on neither conservation nor transitivity. There was one additional student who failed to meet the criterion on only one of each of conservation and transitivity. Thus 2/3 of the pupils could not conserve the relations or use the transitive property at the beginning of the study.

## Conclusions

The smallness of the sample of this study restricts one from justifiably forming generalizations about the ability of EMR children in general to conserve and use the transitive property of the relations, "same number as," "more than," and "fewer than." However, the findings before training are in agreement with previous empirical research. If further replications substantiate these findings, it would appear that older EMR children are not prepared to meaningfully learn operations and relations of numbers, and that learning which has occurred may only be on a basis of rote memorization.

Furthermore, it appeared that the activities used in this study were effective in improving the ability of the students in the sample to use

conservation and the transitive property of the relations. These lessons included a variety of materials and activities, which gave experiences in the concepts of conservation and transitivity. Activities which would have been trivial for normal children of the same age were appealing to the EMR children of this study. The results seem to indicate that it is feasible to train older EMR children on the tasks of this study.

The result that the children in this study retained the ability to conserve and use the transitive property over a period of two to four weeks, gives evidence that the learning that took place may be on a permanent basis. No attempt was made in the present study to determine if the learning transferred to other kinds of relations such as relations on length of segments. Such studies should be carried out.

Finally, this study suggests that further studies should be carried out to properly characterize the cognitive development of older EMR children. These studies should suggest the nature of curriculum materials that can be meaningfully utilized by EMR pupils.

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